Solutions of JEE Advanced-1 | Paper-1 | JEE 2024

PHYSICS

SECTION 1 | MULTIPLE CORRECT ANSWERS TYPE

1.(ABCD) Let the positive X-axis be towards East and the positive Y-axis towards North

Let the velocity of the train be $\vec{v}_T$

Let the initial velocity of the cyclist be $\vec{v}_{C1} = 4\hat{j}$

Let the initial velocity of the train relative to the cyclist be $\vec{v}_{TC1}$

We are given that $\vec{v}_{TC1} = -12\hat{i} + 12\hat{j}$

Now, we know from the definition of relative velocity that

$$\vec{v}_{TC1} = \vec{v}_T - \vec{v}_{C1}$$

Therefore, $\vec{v}_T - \vec{v}_{C1} = -12\hat{i} + 12\hat{j}$

So, $\vec{v}_T = \vec{v}_{C1} + \left(-12\hat{i} + 12\hat{j}\right) = -12\hat{i} + 16\hat{j}$

Hence, the train's actual velocity is 20 m/s, in a direction $\tan^{-1}\left(\frac{12}{16}\right) = \tan^{-1}\left(\frac{3}{4}\right)$ West of North

Now, if the cyclist moves towards East, his velocity is $\vec{v}_{C2} = 4\hat{i}$

Therefore, the velocity of the train as seen by him now is

$$\vec{v}_{TC2} = \vec{v}_T - \vec{v}_{C2} = (-12\hat{i} + 16\hat{j}) - 4\hat{i} = -16\hat{i} + 16\hat{j}$$

The velocity of the train as seen by the cyclist is $\vec{v}_{TC} = \vec{v}_T - \vec{v}_C$

So, it is clear that if he orients his velocity opposite to the train's actual velocity, their vector subtraction has the maximum magnitude

2.(AC) (A) $P = \vec{F}_{ext}.\vec{V}$

Where $\vec{V}$ is the vel. of point of application

$F_{ext} + m_1 g = T$ & $m_2 g = T$

$\Rightarrow \quad F_{ext} = m_2 g - m_1 g$

$= (m_2 - m_1)g$

$\therefore \quad P = (m_2 - m_1)gv$

(B) $F_{ext} + m_1 g - T = m_1 a$

$T - m_2 g = m_2 a$

$F_{ext} = (m_1 + m_2)a + (m_2 - m_1)g$

$= m_2(g + a) - m_1(g - a)$

$\therefore \quad P = (F_{ext})(0 + at)$

$= \{m_2(g + a) - m_1(g - a)\}at$

3.(BCD) $a = g\sin\theta - \mu g\cos\theta$

$= 10\times\frac{3}{5} - 0.5\times10\times\frac{4}{5} = 2\ \text{m/s}^2$

(C) $F_{\min} = mg\sin\theta - \mu mg\cos\theta = 10N$

(D) $mg\sin\theta = \mu(mg\cos\theta + F)$

$30 = 0.5(40+F) \Rightarrow F = 20N$

4.(AB) $\Rightarrow \quad a = -k\sqrt{v}$

$\frac{dv}{dt} = -k\sqrt{u} \qquad \int_{v_0}^{v}\frac{dv}{\sqrt{v}} = -\int_0^t dt$

$\Rightarrow \quad 2(\sqrt{v} - \sqrt{v_0}) = -kt$

$v = 0 \quad \Rightarrow \quad t = \frac{2\sqrt{v_0}}{k} = \text{time of motion}$

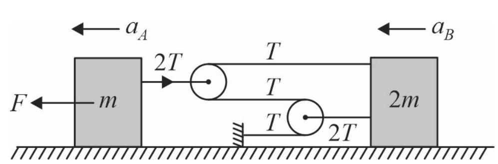
$\Rightarrow \quad \frac{vdv}{dx} = -k\sqrt{v} \quad \Rightarrow \quad \int_{v_0}^{0}\sqrt{v}dv = -k\int_0^s dx; \qquad s = \frac{2v_0^{3/2}}{3k}$

$v_{av} = \frac{s}{t} = \frac{2v_0^{3/2}}{3k}.\frac{k}{2\sqrt{v_0}} = \frac{v_0}{3}$

5.(AC) $2a_A = 3a_B \qquad \ldots (1)$

$F - 2T = ma_A \qquad \ldots (2)$

$3T = 2ma_B \qquad \ldots (3)$



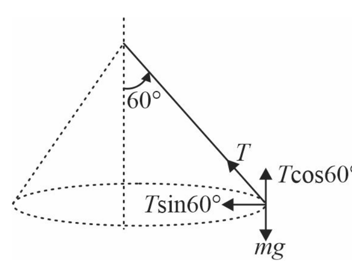
Solving: $a_A = \frac{9F}{17m}$;

$a_B = \frac{6F}{17m}; \qquad a_{rel} = \frac{3F}{17m}$

6.(ABCD) $T\cos60^\circ = mg$

$T\sin60^\circ = m\omega^2\ell\sin60^\circ$

$\cos60^\circ = \frac{g}{\omega^2\ell}; \qquad \omega = \sqrt{\frac{2g}{\ell}}$



Time period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{2g}} \quad = \frac{2\sqrt{2}\pi}{5}$ seconds

Velocity $\qquad v = \ell\sin60^\circ\omega = \sqrt{24}\ \text{m/s}$

$a_c = \omega^2\ell\sin60^\circ = 10\sqrt{3}\ \text{m/s}^2$

SECTION - 2 | MATCHING LIST TYPE

7.(D) (1-p,q), (2-p,r,s), (3-p,s,t), (4-p,s,t)

(1) $|\vec{v}| = \text{const.}$

$|\vec{a}_T| = 0, a = a_R$

$\vec{v} \perp \vec{a}_R \quad \Rightarrow \quad \vec{v}.\vec{r} = 0$

$\vec{v}.\vec{a} = 0$

If $|\vec{a}_T| \neq 0$

(2) v is increasing, $a = \sqrt{a_R^2 + a_T^2}$

$\vec{v}.\vec{r} = 0$

$\vec{v}.\vec{a} > 0$ as $\theta < 90°$

$\vec{r}.\vec{a} < 0$

(3) v is decreasing, $a = \sqrt{a_R^2 + a_T^2}$

$\vec{v}.\vec{r} = 0$

$\vec{v}.\vec{a} < 0$ as $\theta < 90°$

$\vec{r}.\vec{a} < 0$

(4) $v = 5 - 2t$

$a_T = \frac{dv}{dt} = -2$ i.e. speed is decreasing same as (3) above

8.(B) (1-p), (2-r), (3-s), (4-q)

$y = \sqrt{3}x - \frac{5}{4}x^2$

(1) for maxima, $\frac{dy}{dx} = \sqrt{3} - \frac{5}{2}x = 0$

$x = \frac{R}{2} = \frac{2\sqrt{3}}{5} \qquad \therefore \qquad R = \frac{4\sqrt{3}}{5}$

(2) putting $x = \frac{2\sqrt{3}}{5}$ in y, we get $y = H = \frac{6}{5} - \frac{5}{4}\left(\frac{12}{25}\right)$; $H = \frac{3}{5}$

(3) $H = \frac{3}{5} = \frac{uy^2}{2g}$; $u_y^2 = 12$; $u_y = 2\sqrt{3}$; $T = \frac{2u_y}{g} = \frac{2\sqrt{3}}{5}$

(4) $u_{av} = u\cos\alpha = u_x$

$R = \frac{4\sqrt{3}}{5} = \frac{2u_x(2\sqrt{3})}{10}$; $\qquad u_x = 2$

9.(A) (1-p, q, t), (2-s, t), (3-r, s), (4-p, s)

(1) $f = 0.5(2g) = 10N$

$a = \frac{20-10}{2} = 5m/s^2$

T at midpoint is given by $T - 5 = 1\ (5)$

$$T = 10N$$

(2) Acceleration of rope is zero $\Rightarrow$ Net force is zero

$F = mg\sin\theta + \mu mg\cos\theta = 20N$

(3) $F - mg\sin\theta - \mu mg\cos\theta = ma$

Solving $a = 0$

(4) $20 - 2a = 2a$

$a = 0$

Tension at the midpoint

$20 - 1g - T = 0;$ $T = 10N$

10.(C) (1-p, q, s), (2-p, q), (3-p, q, r, t), (4-p, t)

$\Delta U = -W_{\text{cons}}$

$\Delta(KE) = W_{\text{all forces}}$

$\Delta U + \Delta(KE) = W_{\text{all}} - W_{\text{cons}}$

$= W_{\text{non cons}} + W_{ext}$; $\Delta p = F_{ext}\,\Delta t$

SECTION 3 | NUMERICAL VALUE TYPE

1.(1.50) Using the parallelogram law of vector addition,

$$R_1 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60^\circ} = \sqrt{F_1^2 + F_2^2 + F_1F_2}$$

$$R_2 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120^\circ} = \sqrt{F_1^2 + F_2^2 - F_1F_2}$$

Therefore, $\dfrac{F_1^2 + F_2^2 + F_1F_2}{F_1^2 + F_2^2 - F_1F_2} = \left(\dfrac{R_1}{R_2}\right)^2 = \dfrac{19}{7}$ $\Rightarrow$ $\dfrac{\left(\dfrac{F_1}{F_2}\right)^2 + 1 + \dfrac{F_1}{F_2}}{\left(\dfrac{F_1}{F_2}\right)^2 + 1 - \dfrac{F_1}{F_2}} = \dfrac{19}{7}$

Now, let $\dfrac{F_1}{F_2} = K$

Therefore, $\dfrac{K^2 + K + 1}{K^2 - K + 1} = \dfrac{19}{7}$ $\Rightarrow$ $12K^2 - 26K + 12 = 0$

Solving, we get $K = \dfrac{3}{2}$ or $\dfrac{2}{3}$ $\therefore$ $F_1 > F_2 \Rightarrow K > 1 \;\therefore\; K = \dfrac{3}{2} = 1.5$

2.(7.33) Let the acceleration of the ball be a upwards. Then, the acceleration of the rod will be $2a$ downwards. So, w.r.t rod, ball moves up with an acceleration $3a$

$\therefore$ $t = \sqrt{\dfrac{2L}{3a}}$

Let, mass of rod = m, then, mass of ball $= \dfrac{3}{2}m$

$T - \dfrac{3}{2}mg = \dfrac{3}{2}ma$ … (i)

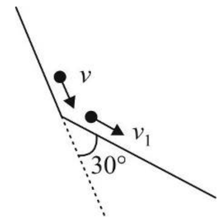
$mg - \dfrac{T}{2} = 2ma$ $\Rightarrow$ $2mg - T = 4ma$ … (ii)

Adding (i) and (ii), we get

$$\frac{mg}{2} = \frac{11}{2}ma \quad \Rightarrow \quad a = \frac{g}{11}$$

Therefore, $t = \sqrt{\dfrac{2L}{3\left(\dfrac{g}{11}\right)}} = \sqrt{\dfrac{22L}{3g}}$

3.(15) Speed just before reaching B is given by energy conservation



$$mg(5) = \frac{1}{2}mv^2 \Rightarrow \ v = \sqrt{2(10)(5)} = 10 \text{ m/s}$$

After collision at B, velocity perpendicular to the incline becomes zero while velocity along the incline remains unchanged

$\therefore \quad v_1 = v\cos 30° = 5\sqrt{3}$ m/s

Velocity upon reaching C can be found by applying energy conservation again.

$$mg(7.5) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\Rightarrow \quad v_2^2 = v_1^2 + 2g(7.5) = 75 + 150 = 225 \quad \therefore \quad v_2 = 15 \text{ m/s}$$

4.(1.40) For no sliding between B and A $m_B\omega^2 r \le \mu_{AB} m_B g$

$$\Rightarrow \quad \omega_{\max} = \sqrt{(0.2)(9.8)} = 1.4 \text{ rad/s}$$

For no sliding between A and table $(m_A + m_B)\omega^2 r \le \mu_{AT}(m_A + m_B)g$

$$\Rightarrow \quad \omega_{\max} = \sqrt{(0.45)(9.8)} = 2.1 \text{ rad/s}$$

$\therefore$ Maximum ω for no sliding at both contacts is 1.4 rad/s

5.(1.75) We know that $v_{MAX}^2 = 2(a)(x) \quad \Rightarrow \quad v_{MAX} = \sqrt{2ax}$

Time taken during the first segment, $T_1 = \dfrac{v_{MAX}}{a} = \sqrt{\dfrac{2x}{a}}$

Time taken during the second segment, $T_2 = \dfrac{x}{v_{MAX}} = \sqrt{\dfrac{x}{2a}}$

Time taken during the third segment, $T_3 = \dfrac{v_{MAX}}{\left(\dfrac{a}{2}\right)} = \sqrt{\dfrac{8x}{a}}$

Total time, $T = T_1 + T_2 + T_3 = \left(\sqrt{2} + \dfrac{1}{\sqrt{2}} + 2\sqrt{2}\right)\sqrt{\dfrac{x}{a}} = \dfrac{7}{\sqrt{2}}\sqrt{\dfrac{x}{a}}$

Distance travelled during the third segment, $X_3 = \dfrac{v_{MAX}^2}{2\left(\dfrac{a}{2}\right)} = 2x$

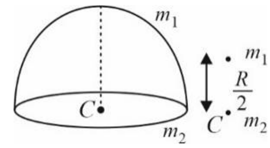
Total distance, $X = x + x + 2x = 4x$

Therefore, $v_{AVG} = \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{X}{T} = \dfrac{4}{7}\sqrt{2ax}$

So, $\dfrac{v_{MAX}}{v_{AVG}} = \dfrac{7}{4} = 1.75$

6.(0.33) Let σ be the mass per unit area (it will be same for the shell and the plate)

$$m_1 = \sigma(2\pi R^2), m_2 = \sigma(\pi R^2)$$



To find the CM, we can replace the hemispherical shell and the circular plate by point mass located at their CM as shown.

CM of the system from C

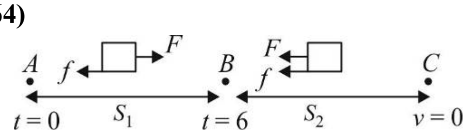
$$= \frac{m_1(R/2) + m_2(0)}{m_1 + m_2} = \frac{\sigma(2\pi R^2)(R/2)}{\sigma 3\pi R^2} = \frac{R}{3} = 0.33R$$

7.(3.46) $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

Here x = 3m, $R = 3 + 6 = 9m$, $\tan\theta = \sqrt{3} = 1.73$

$$h = 2\sqrt{3} = 3.46m$$

8.(34.64)



From $A(t=0)$ to $B(t=6)$

$$a = \frac{F - f}{m} = 10^{-5} = 5 \text{ m/s}^2$$

$$s_1 = \frac{1}{2}at^2 = \frac{1}{2}(5)(36) = 90m$$

Velocity at $B = at = 5(6) = 30$ m/s

After B, the direction of F is reversed.

As velocity is in forward direction, friction acts backwards till block comes to instantaneous rest at C.

From B to C

$$a = \frac{(F - f)}{m} = -(10 + 5) = -15 \text{ m/s}^2$$

$$u = 30 \text{ m/s}$$

$$v^2 = u^2 + 2as_2$$

$$0 = (30)^2 + 2(-15)s_2 \Rightarrow s_2 = 30m$$

While returning from C to A, F acts towards left but friction acts towards right (as velocity is towards left)

$$s = -(s_1 + s_2) = -120m$$

$$a = -\frac{(F - f)}{m} = -5 \text{ m/s}^2$$

$$v^2 = u^2 + 2as = 0 + 2(-120)(-5)$$

$$v^2 = 1200 \Rightarrow v = 20\sqrt{3} = 20(1.732) = 34.64 \text{ m/s}$$

CHEMISTRY

SECTION 1 | MULTIPLE CORRECT ANSWERS TYPE

1.(ABCD)

For reversible isothermal expansion T is constant hence, PV = Constant,

During isothermal expansion entropy increases.

$w = -[n R T \ln V_2 - n R T \ln V_1]$

2.(AB) HBr is less polar than HF

Order of dipole moment is $H_2S < HF < H_2O$

CuCl is more covalent than NaCl

3.(ABC)

F is most electronegative elements and Cl has maximum value of electron affinity. Cl can expand octet due to the presence of vacant d orbitals. Both HOF and HOCl are acidic compounds.

4.(ACD)

For mixing of two ideal gases at constant T and P, ΔU_{mix} and ΔH_{mix} are zero. During mixing, entropy of system increase while $q_{mix} = 0$.

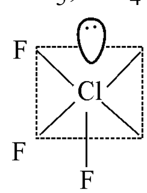
5.(A) If $\Delta n_g = 0$ then there is no effect of change in pressure on equilibrium state.

6.(CD) Given reaction is spontaneous when $T > \frac{\Delta H}{\Delta S}$

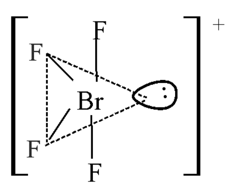
$$T > \frac{61.17 \times 10^3}{132} \quad \Rightarrow \quad T > 463.4\,K$$

SECTION - 2 | MATCHING LIST TYPE

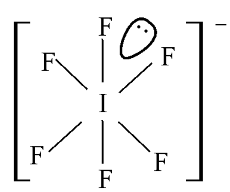
7.(D) **(1)** ClF_5, BrF_4^+, IF_6^- all have same oxidation state (+5)



(square pyramidal)



(see-saw)

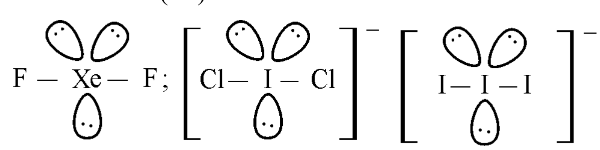


(distored octahedral)

All have one lone pair of electrons each; but different shapes ; $\mu \neq 0$ so polar.

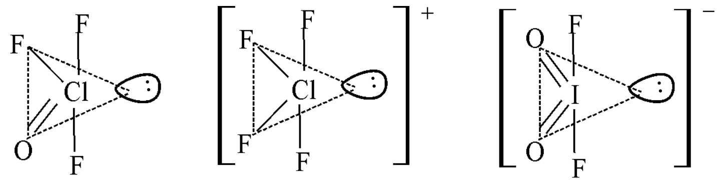
(2) ClF_3, BrF_2^+, ICl_4^- all have same oxidation state (+3)

(3) $XeF_2 = +2$; $ICl_2^- = +1$ $I_3^- = +1$;



All have three lone pairs each and same shape but different oxidation state. In all $\mu = 0$; so non-polar

(4) $ClOF_3$, ClF_4^+, $IO_2F_2^-$ all have same oxidation number (+5)



see-saw see-saw see-saw

In all $\mu \neq 0,$ so all polar

8.(A) **(1)** For isoelectronic species, the ionic size decreases with increase in nuclear charge. Hydration $\propto$ charge on anion and heavier hydrated ions move slowly. So, it is not correct order.

(2) Heavier hydrated ions move slowly. Number of atomic shells increases, ionic size increases.

(3) Correct order; as Cl has less inter electronic repulsions than F due to bigger size of 3p-subshell.

(4) Oxidation state increases, the electronegativity increase. For isoelectronic species, ionisation energy and electron affinity increases with increasing nuclear charge.

9.(B) **(1)** $6 \rightarrow 3 \quad \Delta n = 3$

$\therefore \quad$ Number of lines $= \dfrac{3(3+1)}{2} = 6$

All lines are in infrared region.

(2) $7 \rightarrow 3 \quad \Delta n = 4$

$\therefore \quad$ Number of line $= \dfrac{4(4+1)}{2} = 10$

All lines are in infrared region.

(3) $5 \rightarrow 2 \quad \Delta n = 3$

All lines are in visible region.

(4) $6 \rightarrow 2 \quad \Delta n = 4$

All lines are in visible region.

10.(C) **(1)** $CO_2(g) + C(s) \longrightarrow 2CO(g) \qquad \Delta_r H^0 = 2(-220) - (-394)$ and $\Delta_r S^0 > 0$

$\Rightarrow \quad \Delta_r G^0 < 0$ and $\Delta_r H^0 - \Delta_r U^0 = \Delta_r U^0 = \Delta n_g RT = (2-1)RT > 0$

(2) $SO_2Cl_2(g) \longrightarrow SO_2(g) + Cl_2(g) \quad \Delta_r S > 0$

and $\Delta_r H - \Delta_r U - \Delta n_g RT = (2-1)RT > 0$

(3) $CO(g) + Cl(g) \longrightarrow COCl_2(g) \qquad \Delta_r S < 0$ and $\Delta_r H - \Delta_r U = (2-1)RT < 0$

(4) $Cl(g) \longrightarrow 2Cl(g) \qquad \Delta_r S > 0$ and $\Delta_r H - \Delta_r H - \Delta_r U = (2-1)RT > 0$

SECTION 3 | NUMERICAL VALUE TYPE

1.(3061.83)

$H_2(g) \rightarrow 2H(g) \quad : \quad E = 435.8$ kJ

$2H(g) \rightarrow 2H^+(g) + 2e^- \quad : \quad E = 2 \times (2.18 \times 10^{-18}) \times 6.023 \times 10^{23} \times 10^{-3}$ kJ $= 2626.028$ kJ

Total energy required = 435.8 + 2626.028 = 3061.828 = 3061.83 kJ

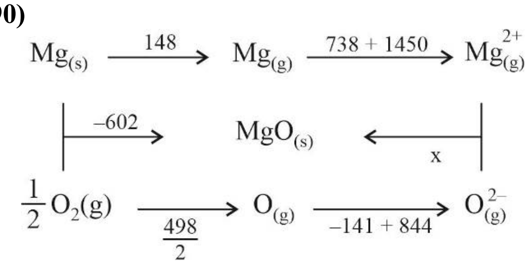
2.(935) $\Delta_r H^0 = -394 - (-581) = 187$ kJ

$\Delta_r S^0 = (210 + 52) - (56 + 6) = 200$ J = 0.2 kJ

The reduction of cassiterite by coke is spontaneous at T greater than T_{switch}

$T_{switch} = \Delta_r H^0 / \Delta_r S^0 = 935$ K

3.(3890)



$$-602 = 148 + 738 + 1450 - x + \frac{498}{2} - 141 + 844 \quad \Rightarrow \quad x = 3890 \text{ kJ / mol}$$

4.(0.38) $K_c = \frac{[z]}{[x]\times[y]} = \frac{0.950}{1.20\times 2.10} = 0.3769 = 0.38$

5.(75) Mol of $CaCl_2$ = Mol of $CaCO_3$ = Mol of CaO $= \frac{0.56}{56} = 0.01$

Mass of $CaCl_2$ = 0.01 × 111 = 1.11
Mass of NaCl = 4.44 – 1.11 = 3.33

% of NaCl $= \frac{3.33}{4.44}\times 100 = 75$

6.(15.66)

Calculated dipole moment = 2.88×10^{-29} C m

Observed dipole moment = $1.35 \times 3.34 \times 10^{-30}$ C m = 4.509×10^{-30} C m

% ionic character = $\frac{4.509}{2.88} \times 10 = 15.65625 = 15.66$

7.(3.44)

$$\frac{P1\times V1}{T1} = \frac{P2\times V2}{T2} \quad \Rightarrow \quad V2 = \frac{P1\times V1\times T2}{P2\times T1} = \frac{650\times 5.50\times 283}{980\times 300} = 3.44$$

8.(80) $CuS \longrightarrow Cu^{2+} + SO_2$ $Cu_2S \longrightarrow Cu^{2+} + SO_2$

n.f. = 6   n.f. = 8

x mol   y mol

$$(eq)_{CuS} + (eq)_{Cu_2S} + (eq)_{Fe^{2+}} = (eq)_{MnO_4^-}$$

$$(x\times 6) + (y\times 8) + \left(\frac{1\times 1\times 200}{1000}\right) = \left(\frac{0.4\times 5\times 400}{1000}\right)$$

$$6x + 8y = 0.6 \quad \Rightarrow \quad 3x + 4y = 0.3 \quad \ldots\ldots \text{(i)}$$

$$96x + 160y = 10 \quad \Rightarrow \quad 9.6x + 16y = 1 \quad \ldots\ldots \text{(ii)}$$

By solving equation (i) and (ii)

$$x = \frac{1}{12},\ y = \frac{1}{80}; \qquad \%\ CuS = \frac{\left(\frac{1}{12}\times 96\right)}{10}\times 100 = 80\%$$

MATHEMATICS

SECTION 1 | MULTIPLE CORRECT ANSWERS TYPE

1.(ABC) $\sin^2 x + 2\sin x + \frac{11}{4} \geq 2$

$\sin x \geq -\frac{1}{2}$

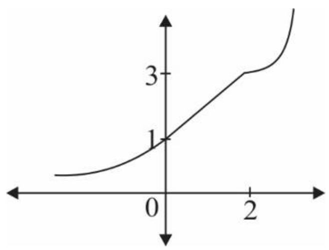
2.(ABD) $a < p \Rightarrow \frac{a}{p} < 1$, limit = 0

$a > p, \frac{a}{p} > 1$, limit $\to \infty$

$a = p, l = 0$

$$\lim_{x\to\infty}\left(\frac{ax^2+bx+c}{px^2+qx+r}\right)^{mx+n} = e^{(b-q)m/p}$$

3.(ABC) $f(0^-) = f(0^+) \Rightarrow \lambda = 1$



4.(ACD) (A) $\frac{a}{1-2a}, \frac{b}{1-2b}, \frac{c}{1-2c}$ are in H.P. $\Leftrightarrow$ $\frac{1-2a}{a}, \frac{1-2b}{b}, \frac{1-2c}{c}$ are in A.P.

$\Leftrightarrow$ $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. $\Leftrightarrow$ a, b, c are in H.P.

(B) $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ are in G.P. $\Leftrightarrow$ $\ln\left(a-\frac{b}{2}\right), \ln\frac{b}{2}, \ln\left(c-\frac{b}{2}\right)$ are in A.P.

(D) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. $\Leftrightarrow$ $e^{1/a}, e^{1/b}, e^{1/c}$ are in G.P.

5.(AC) $(x+a)(x+1991)+1=0$

$\Rightarrow$ $(x+a)(x+1991) = -1$ $\Rightarrow$ $(x+a) = 1$ *and* $x + 1991 = -1$

$\Rightarrow$ $a = 1993$ or $x + a = -1$ and $x + 1991 = 1$ $\Rightarrow$ $a = 1989$

6.(AB) $\because$ $\sin C = \frac{1-\cos A\cos B}{\sin A\sin B} \leq 1$ $\Rightarrow$ $\cos(A-B) \geq 1$

$\Rightarrow$ $\cos(A-B) = 1 \Rightarrow$ $A - B = 0$ $\Rightarrow$ $A = B$ $\therefore$

$\sin C = \frac{1-\cos^2 A}{\sin^2 A} = 1$ $\Rightarrow$ $C = 90°$

SECTION - 2 | MATCHING LIST TYPE

7.(A) **(P)** $\cot x < 0 \Rightarrow -\cot x = \cot x + \text{cosec } x$

$\Rightarrow \quad 2\cos x + 1 = 0 \quad \Rightarrow \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$

But $\quad \cot\frac{4\pi}{3} > 0$

(Note: $\cot x \geq 0$ is not possible)

Hence, $x = \frac{2\pi}{3}$ only satisfies above equation.

(Q) The curves $y = n|\sin x|$ and $y = m|\cos x|$ intersect at 4 points in $[0, 2\pi]$.

(R) The given equation is valid if $\frac{9x}{10\pi}$ is an integer.

$x = 10\pi, \frac{100\pi}{9} \quad \Rightarrow \quad x = \frac{110\pi}{9}$

Hence, only one solution

(S) $\frac{\sqrt{3}-1}{2\sqrt{2}\sin x} + \frac{\sqrt{3}+1}{2\sqrt{2}\cos x} = 2$

$\sin\frac{\pi}{12}\cos x + \cos\frac{\pi}{12}\sin x = \sin 2x$

$\sin 2x = \sin\left(x + \frac{\pi}{12}\right) \quad \therefore \quad 2x = x + \frac{\pi}{12} \text{ or } 2x = \pi - \left(x + \frac{\pi}{12}\right)$

$x = \frac{\pi}{12} \text{ or } 3x = \frac{11\pi}{12} \quad \therefore \quad x = \frac{\pi}{12} \text{ or } \frac{11\pi}{36}$

Therefore, k is 12

8.(C) **(1)** $\cos(p\sin x) = \sin(p\cos x) = \cos\left(\frac{\pi}{2} - p\cos x\right)$

$\Rightarrow \quad p\sin x = \frac{\pi}{2} - p\cos x \Rightarrow p(\sin x + \cos x) = \frac{\pi}{2}$

$\sin x + \cos x = \frac{\pi}{2p} \leq \sqrt{2} \Rightarrow \frac{4\sqrt{2}p}{\pi} \geq 2$

(2) $9^3 3^{3\cos 2x + 4\sin 2x}$

$\because \quad 3\cos 2x + 4\sin 2x \geq -5 \quad \therefore \quad 3^{3\cos 2x + 4\sin 2x} \geq 3^{-5} \quad \therefore \quad 3^6 . 3^{3\cos 2x + 4\sin 2x} \geq 3$

(3) $\tan 40° + \tan 10° + \tan 10°$

$= \frac{\sin 40°\cos 10° + \cos 40°\sin 10°}{\cos 10°\cos 40°} + \frac{\sin 10°}{\cos 10°} = \frac{1}{\cos 10°} + \frac{\sin 10°}{\cos 10°} = \frac{1+\cos 80°}{\sin 80°} = \cot 40°$

$\therefore \quad k = 1 \qquad \therefore \quad 10 - k = 9$

(4) Both roots of $x^2 + 5x + 1 = 0$ are negative

$\therefore \quad \lambda < 0$

$\tan^{-1}\left(\frac{1}{\lambda}\right) = \cot^{-1}(\lambda) - \pi$

$\therefore \quad \tan^{-1}\lambda + \cot^{-1}\lambda - \pi = -\frac{\pi}{2} = k\pi \quad \therefore \quad k = -\frac{1}{2} \quad \therefore \quad \frac{2}{k} + 5 = 1$

9.(A) (1) $\to$ (p, q, s, t), (2) $\to$ (q), (3) $\to$ (p, q, s), (4) $\to$ (p, s)

(1) $f(x)=\sin^2 2x-2\sin^2 x=2\sin^2 x\cos 2x$

Function is even, hence many one, function is also periodic.

$$f(x)=(1-\cos 2x)\cos 2x=\frac{1}{4}-\left(\cos 2x-\frac{1}{2}\right)^2$$

Range of function is $\left[-2,\frac{1}{4}\right]$.

(2) $f(x)=4x$

(3) $f(x)=\sqrt{\ln(\cos(\sin x))}$

$\ln(\cos(\sin x))\ge 0 \quad\Rightarrow\quad \cos(\sin x)=1$

$\Rightarrow \quad f(x)=0$

(4) $f(x)=\tan^{-1}\left(\dfrac{x^2+1}{x^2+\sqrt{3}}\right)$

$f(x)$ is even and hence many one.

Range is $\left[\dfrac{\pi}{6},\dfrac{\pi}{4}\right)$.

10.(A) (1) $\to$ (r), (2) $\to$ (p), (3) $\to$ (q), (4) $\to$ (s)

(1) a, b, c are in A.P.

$b-a=c-b$

$b-a, c-b, a$ are in G.P.

$\dfrac{c-b}{b-a}=\dfrac{a}{c-b}\Rightarrow c-b=a \quad (\because\ b-a=c-b)$

(2) a, x, b are in A.P.

$x=\dfrac{a+b}{2}$

a, y, z, b are in G.P.

$y=a^{2/3}b^{1/3}, z=a^{1/3}b^{2/3}$

(3) $a, b=ar, c=ar^2$

If $c>4b-3a$

$r^2-4r+3>0 \quad (\because a>0)$

$(r-3)(r-1)>0$

(4) $7x^2-8x+9<0$

A = 7 > 0, D = 64 – 252 < 0

No solution.

SECTION 3 | NUMERICAL VALUE TYPE

1.(2015) Let $\dfrac{\theta}{2^{n-1}}=A \quad\Rightarrow\quad \dfrac{2\theta}{2^n}=A \quad\Rightarrow\quad \dfrac{\theta}{2^n}=\dfrac{A}{2}$

$$\Rightarrow \quad \tan\left(\frac{\theta}{2^n}\right)\sec\left(\frac{\theta}{2^{n-1}}\right)=\tan\frac{A}{2}\cdot\sec A=\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}.\cos A}$$

$$=\frac{\sin\left(A-\frac{A}{2}\right)}{\cos\frac{A}{2}\cdot\cos A}=\frac{\sin A.\cos\frac{A}{2}-\cos A\cdot\sin\frac{A}{2}}{\cos\frac{A}{2}\cdot\cos A}=\tan A-\tan\frac{A}{2}$$

$$\Rightarrow \quad \tan\frac{\theta}{2^n}\cdot\sec\frac{\theta}{2^{n-1}} = \left(\tan\frac{\theta}{2^{n-1}} - \tan\frac{\theta}{2^n}\right)$$

$$\sum_{n=1}^{2015}\tan\frac{\theta}{2^n}\sec\frac{\theta}{2^{n-1}} = \sum_{n=1}^{2015}\left[\tan\left(\frac{\theta}{2^{n-1}}\right) - \tan\frac{\theta}{2^n}\right]$$

$$= \tan\left(\frac{\theta}{2^0}\right) - \tan\left(\frac{\theta}{2^1}\right) + \tan\left(\frac{\theta}{2^1}\right) - \tan\left(\frac{\theta}{2^2}\right) + \tan\left(\frac{\theta}{2^2}\right) - \tan\left(\frac{\theta}{2^3}\right) + \ldots + \tan\left(\frac{\theta}{2^{2014}}\right) - \tan\left(\frac{\theta}{2^{2015}}\right)$$

$$= \tan\left(\frac{\theta}{2^0}\right) - \tan\left(\frac{\theta}{2^{2015}}\right) \quad \Rightarrow \quad a = 0\ \&\ b = 2015 \ \Rightarrow\ a + b = 2015.$$

2.(1.27) $\because \quad \frac{1}{4}\sin 3\theta = \frac{27}{4}\sin\left(\frac{\theta}{9}\right) - \sin^3\theta - 3\sin^3\frac{\theta}{3} - 9\sin^3\left(\frac{\theta}{9}\right)$

$$\Rightarrow \quad \sin 3\theta = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \text{ or } \sin\frac{3\pi}{4} \qquad \Rightarrow \quad \theta = \frac{\pi}{12} \text{ or } \frac{\pi}{4}$$

$$\therefore \quad \tan\frac{\pi}{12} = 2-\sqrt{3} \text{ or } \tan\frac{\pi}{4} = 1 \qquad \therefore \quad \tan\frac{\pi}{12} + \tan\frac{\pi}{4} = 3-\sqrt{3}$$

3.(0.67) $T_r = \dfrac{\frac{r(r+1)(2r+1)}{6}}{\frac{r^2(r+1)^2}{4}} = \dfrac{2r+1}{6}\times\dfrac{4}{r(r+1)} = \dfrac{2}{3}\dfrac{2r+1}{r(r+1)}$

$$T_r = \frac{2}{3}\left[\frac{1}{r} + \frac{1}{r+1}\right]$$

$$S_n = -T_1 + T_2 - T_3 + T_4 \ldots\ldots$$

$$= \frac{2}{3}\left[\frac{-1}{1} \frac{-1}{2} + \frac{1}{2} + \frac{1}{3} - \frac{1}{3} - \frac{1}{4} + \frac{1}{4} \ldots\ldots\right] \qquad \therefore \quad \lim_{n\to\infty} S_n = \frac{-2}{3} \qquad \therefore \quad \lim_{n\to\infty} |S_n| = \frac{2}{3}$$

4.(0.50) $e^{f(x)} = \dfrac{10+x}{10-x},\ x \in (-10, 10)$

$$f(x) = \log\left(\frac{10+x}{10-x}\right)$$

$$f\left(\frac{200x}{100+x^2}\right) = \log\left[\frac{10 + \frac{200x}{100+x^2}}{10 - \frac{200x}{100+x^2}}\right] = \log\left[\frac{10(10+x)}{10(10-x)}\right]^2 = 2\log\left(\frac{10+x}{10-x}\right) = 2f(x)$$

$$f(x) = \frac{1}{2}f\left(\frac{200x}{100+x^2}\right) \Rightarrow k = \frac{1}{2} = 0.5$$

5.(43) Since $f(x)$ has its domain as R

$$\therefore \quad a - 3 = 0 \quad \Rightarrow \quad a = 3$$

$$f(x) = \frac{bx}{x^2+4} = y$$

$$bx = x^2y + 4y \quad \Rightarrow \quad x^2y - bx + 4y = 0$$

$\because \quad x$ is real

$\therefore \quad D \geq 0$

$b^2 - 16y^2 \geq 0 \quad \Rightarrow \quad (4y-b)(4y+b) \leq 0$

$y \in \left[\frac{-b}{4}, \frac{b}{4}\right] \equiv [-8, 8]$

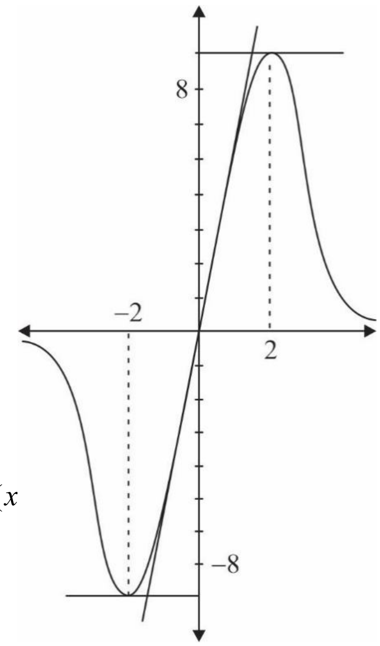
$\frac{b}{4} = 8 \quad \Rightarrow \quad b = 32$

$f(x) = \frac{32x}{x^2+4}$

$f'(x) = 32\left(\frac{(x^2+4)\cdot 1 - x \cdot 2x}{(x^2+4)^2}\right) = \frac{32(4-x^2)}{(x^2+4)^2}$

$f'(x) > 0 \quad \Rightarrow \quad x \in (-2, 2) \quad \Rightarrow \quad f(x)\uparrow$

$f'(x) < 0 \quad \Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty) \quad \Rightarrow \quad f(x$



Slope of the tangent at origin

$\left.\frac{dy}{dx}\right|_{x=0} = 32 \times \left(\frac{4}{16}\right) = 8$

Now, equation $f(x) = mx$ gives three solutions is $m \in (0, 8)$

$\therefore \quad a + b + p + q = 3 + 32 + 0 + 8 = 43$

6.(5.67) $a = 3, b = 12, c = 9$

$$\lim_{x \to 0} \frac{ax\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} \ldots\ldots\right)}{x^2\left(x - \frac{x^3}{6}\right)}$$

$$-b\left(x - \frac{x^2}{2} + \frac{x^3}{3} \ldots\right)$$

$$\frac{+c.x.\left(1 - x + \frac{x^2}{2} \ldots\ldots\right)}{x^2\left(x - \frac{x^3}{6} + \ldots\right)} = 2$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{x(a-b+c) + x^2\left(a + \frac{b}{2} - c\right) + x^3\left(\frac{a}{2} - \frac{b}{3} + \frac{c}{2}\right) + \ldots\ldots}{x^3} = 2$$

$a - b + c = 0 \qquad a + \frac{b}{2} - c = 0$

$\frac{a}{2} - \frac{b}{3} + \frac{c}{2} = 2 \qquad$ Solving we get $a = 3, b = 12, c = 9$

7.(20) e^{-1}

$$\text{Limit} = \lim_{x \to 0}\left(1 - \frac{x - \sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$$

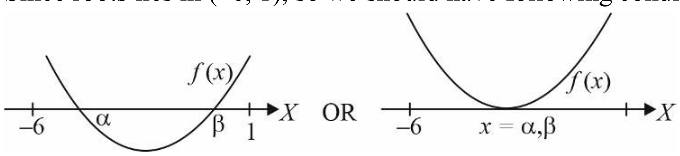
$$= \lim_{x \to 0} \left\{ \left(1 + \frac{-1}{\left(\frac{x}{x - \sin x} \right)} \right)^{\frac{x}{x - \sin x}} \right\}^{\frac{\sin x}{x}} = \left\{ \lim_{y \to \infty} \left(1 + \frac{-1}{y} \right)^{y} \right\}^{\lim_{x \to 0} \frac{\sin x}{x}}$$

$$\left\{ \text{putting} \frac{x}{x - \sin x} = y \to \infty \,\text{as}\, x \to 0 \right\}$$

$$= \left(e^{-1}\right)^{1} = e^{-1} .$$

8.(12) Let $f(x) = x^2 + 2(p-3)x + 9$

Since roots lies in (–6, 1), so we should have following conditions.



(i) $D \geq 0 \Rightarrow 4(p-3)^2 - 36 \geq 0$

$\Rightarrow p(p-6) \geq 0$

$\Rightarrow p \leq 0$ or $p \geq 6$ … (1)

(ii) $f(-6) > 0 \Rightarrow p < \frac{27}{4}$ … (2)

(iii) $f(1) > 0 \Rightarrow p > -2$ … (3)

(iv) $-6 < \frac{\alpha + \beta}{2} < 1 \Rightarrow 2 < p < 9$ … (4)

$\therefore$ From (i), (ii), (iii) & (iv),

We get $6 \leq p < \frac{27}{4}$

$\therefore$ integral value of $'p' = 6$

Since $2, g_1, g_2, g_3, \ldots\ldots, g_{17}, g_{18}, g_{19}, g_{20}, 6$ are in G.P.

$\therefore$ $g_4 g_{17} = 2 \times 6 = 12$